

Comment on “Topological stability of the half-vortices in spinor exciton-polariton condensates”

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We show that the conclusions of recent paper by Flayac *et al.* [*Phys. Rev. B* **81**, 045318 (2010)] concerning the stability of half-quantum vortices are misleading. We demonstrate the existence of static half-quantum vortices in exciton-polariton condensates and calculate the warping of their texture produced by transverse-electric-transverse-magnetic splitting of polariton band.

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Half-quantum vortices are topological excitations of multicomponent condensates with combined spin-gauge symmetry.¹ They have been discussed² to be the lowest energy topological excitations in exciton-polariton condensates in semiconductor microcavities with unpinned linear polarization of the condensate. The half-quantum vortices are expected to define the Berezinskii-Kosterlitz-Thouless (BKT) transition in this system,^{2,3} and they have been recently observed experimentally.⁴

In a recent paper⁵ Flayac *et al.* studied the effects of transverse-electric-transverse-magnetic (TE-TM) splitting of the exciton-polariton band on the state of vortices. The authors of Ref. 5 have not found the static half-vortex solutions of quasiequilibrium two-component Gross-Pitaevskii equation and concluded that “the half-vortices are no more stationary solutions of the spinor Gross-Pitaevskii equations and should not affect the critical temperature of the BKT phase transition.” The goals of this Comment are to show that this conclusion is incorrect, to indicate the mathematical error of Ref. 5 that prevented the authors to establish the stationary half-vortex solutions, and to present the correct way of solving this problem.

The order parameter of exciton-polariton condensate is the two-dimensional complex vector $\boldsymbol{\psi}$. This vector can be written in terms of two circular-polarization components, ψ_{\pm} , as

$$\boldsymbol{\psi} = \frac{\hat{x} + i\hat{y}}{\sqrt{2}} \psi_+ + \frac{\hat{x} - i\hat{y}}{\sqrt{2}} \psi_- \quad (1)$$

For the polariton condensate in quasiequilibrium, the components of the order parameter ψ_{\pm} satisfy two coupled Gross-Pitaevskii equation [Eq. (10) of Ref. 5]. In the case when TE-TM splitting is present, the static vortex solutions can be found numerically, but one has to use the correct asymptotic behavior of ψ_{\pm} . In Ref. 5 it was assumed that the variables can be separated and, moreover, that the phases of ψ_{\pm} change linearly with the azimuthal angle [see Eq. (12) of Ref. 5]. These assumptions are incorrect, and due to this incorrect substitution (12) the static half-vortex solutions were not found in Ref. 5. In fact, we show below that *the circular phases of the condensate order parameter are nonlinear functions of the azimuthal angle.*

To find the asymptotic behavior of static solutions ψ_{\pm} at large distances r from the half-vortex core (i.e., in the elastic region), we note that in this region the polarization of the

condensate becomes linear and the amplitudes of the components of the order parameter becomes equal, $|\psi_{\pm}| = \sqrt{n/2}$, where $n \equiv n(\infty)$ is the concentration of the uniform condensate. Therefore, the asymptotics of ψ_{\pm} can be written as

$$\psi_{\pm}(r \rightarrow \infty, \phi) = \sqrt{\frac{n}{2}} e^{i[\theta(\phi) \mp \eta(\phi)]}. \quad (2)$$

Here we denoted the circular phases as $\theta \mp \eta$, using the common phase angle θ and the polarization angle η , as defined in Ref. 2. These angles are yet unknown functions of the azimuthal angle ϕ .⁶

To calculate the functions $\eta(\phi)$ and $\theta(\phi)$ we first obtain the elastic energy of the condensate in the presence of TE-TM splitting. Substitution of Eq. (2) into the quasiequilibrium Hamiltonian of polariton condensate [Eqs. (3)–(8) of Ref. 5] gives

$$H_{\text{el}} = \frac{\hbar^2 n}{2m^*} \int dx dy \left\{ (\nabla \eta)^2 + (\nabla \theta)^2 + 2\gamma \left[\left(\frac{\partial e^{-i(\theta-\eta)}}{\partial z} \right) \times \left(\frac{\partial e^{i(\theta+\eta)}}{\partial z} \right) + \left(\frac{\partial e^{i(\theta-\eta)}}{\partial z^*} \right) \left(\frac{\partial e^{-i(\theta+\eta)}}{\partial z^*} \right) \right] \right\}. \quad (3)$$

Here the effective mass m^* and the TE-TM splitting parameter γ are defined by

$$\frac{1}{m^*} = \frac{1}{2} \left(\frac{1}{m_l} + \frac{1}{m_t} \right), \quad \gamma = \frac{m_t - m_l}{m_t + m_l}, \quad (4)$$

where the effective mass of transverse (or the TE) polaritons is m_t and the effective mass of longitudinal (or the TM) polaritons is m_l . In Eq. (3) we also use the complex derivative

$$\frac{\partial}{\partial z} = \frac{1}{2} \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right). \quad (5)$$

The equations for the angles θ and η are obtained by variation of the functional H_{el} . In the limit $r \rightarrow \infty$ only azimuthal derivatives are to be kept in these equations and we obtain

$$[1 - \gamma \cos(2u)]\theta' + 2\gamma \sin(2u)u'\theta' = 0, \quad (6a)$$

$$[1 + \gamma \cos(2u)]u'' + \gamma \sin(2u)(1 - u'^2 - \theta'^2) = 0, \quad (6b)$$

where

$$u(\phi) = \eta(\phi) - \phi. \quad (7)$$

We note that the superfluid current \mathbf{J} that corresponds to the Hamiltonian density, Eq. (3), is given by more complicated expression than the usual one. In particular, for the wave function (2) the radial and azimuthal components of the current are

$$J_r = \frac{\hbar n}{m^* r} \gamma \sin(2u) \frac{d\theta}{d\phi}, \quad (8a)$$

$$J_\phi = \frac{\hbar n}{m^* r} [1 - \gamma \cos(2u)] \frac{d\theta}{d\phi}. \quad (8b)$$

The equation of continuity of the current for the static solutions, $\text{div } \mathbf{J}=0$, is reduced to the condition $dJ_\phi/d\phi=0$ and gives Eq. (6a) again.

In what follows we denote the vortex solutions as (k, m) with the polarization and phase winding numbers k and m , respectively. These numbers are defined by $\eta(2\pi) - \eta(0) = 2\pi k$ and $\theta(2\pi) - \theta(0) = 2\pi m$, and they can be integer or half integer provided the sum $k+m$ is an integer.² The topological charges used in Ref. 5 are $l_\pm = m \mp k$. Equations (6a) and (6b) have simple solutions for $k=1$ and an arbitrary integer m . Namely, $\eta = \phi$ and $\theta = m\phi$ so that $u(\phi) \equiv 0$. Only these solutions with $l_- - l_+ = 2$ were claimed to exist and were analyzed in Ref. 5. The variables are indeed separated in this case. Moreover, the polarization field for the (1,0) vortex (hedgehog) is purely longitudinal,⁷ and it is described by the radial function of the usual vortex in one-component condensate with the longitudinal mass m_l .

Apart from trivial solutions, Eqs. (6a) and (6b) can be solved for any other winding numbers (k, m) . Interestingly, there are two qualitatively distinct type of solutions for elementary half-quantum vortices with $k, m = \pm 1/2$. In real microcavities the TE-TM splitting is small and we first present the series of these solutions in the powers of $\gamma \ll 1$. For the $(1/2, \pm 1/2)$ half-vortices the solutions are

$$\theta(\phi) = \pm \left[\frac{1}{2} \phi + \frac{\gamma}{2} \sin(\phi) + \frac{\gamma^2}{4} \sin(2\phi) + \dots \right], \quad (9a)$$

$$\eta(\phi) = \frac{1}{2} \phi - \frac{\gamma}{2} \sin(\phi) + \frac{\gamma^2}{8} \sin(2\phi) + \dots. \quad (9b)$$

For the $(-1/2, \pm 1/2)$ half-vortices the solutions are

$$\theta(\phi) = \pm \left[\frac{1}{2} \phi + \frac{\gamma}{6} \sin(3\phi) + \frac{\gamma^2}{36} \sin(6\phi) + \dots \right], \quad (10a)$$

$$\eta(\phi) = -\frac{1}{2} \phi + \frac{\gamma}{6} \sin(3\phi) - \frac{\gamma^2}{24} \sin(6\phi) + \dots. \quad (10b)$$

The solutions to Eqs. (6a) and (6b) can also be found

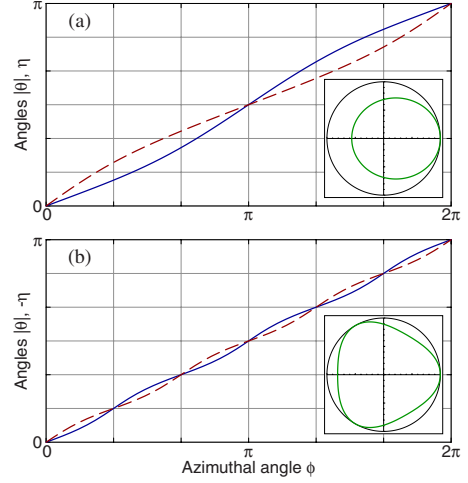


FIG. 1. (Color online) Showing the polarization angle $\eta(\phi)$ (solid blue line) and the phase angle $\theta(\phi)$ (dashed red line) of the order parameter of half-quantum vortices. (a) The case of $(1/2, \pm 1/2)$ half-vortices. (b) The case of $(-1/2, \pm 1/2)$ half-vortices. Insets show the warping of streamlines (thick green lines) of the current around the half-vortex core with respect to the perfect circles (thin black lines).

numerically as shown in Fig. 1. We have used a high value of the TE-TM splitting parameter, $\gamma=0.3$, in order to illustrate the qualitative features of the behavior of polarization and phase angles. Analyzing the superfluid current around the vortex core we note that the streamlines are warped with respect to perfect circles. Physically, the warping appears due to the change in the polariton mass with polarization. Using Eqs. (8a) and (8b) we find the streamlines to be defined by equation

$$\frac{d \ln r}{d\phi} = \frac{\gamma \sin[2u(\phi)]}{1 - \gamma \cos[2u(\phi)]}. \quad (11)$$

The warping of streamlines is shown in the insets of Fig. 1.

The asymptotics found above divide the solutions of the Gross-Pitaevskii equation into topologically distinct classes according to the values of winding numbers k and m . The half-vortex should be found by minimizing the full Gross-Pitaevskii Hamiltonian within a particular topological class. This solution exists since the energy is bound from below. This solution is static since it is a minimum of the Hamiltonian. The half-vortices can be found either by numerical solution of Gross-Pitaevskii equation with the required asymptotic behavior or by other means (e.g., by variational method).

Finally, we comment on the stability of half-vortices from the other point of view. In the half-vortex core, when $r \rightarrow 0$, one of the circular components goes to zero and is singular: the order parameter behaves as $re^{\pm i\phi}$ so that the gradient is not defined at $r=0$. For example, for the $(1/2, 1/2)$ half-vortex, $\psi_- \propto re^{i\phi}$ and $\psi_+ = \text{const}$.⁸ If a half-vortex state is created by some external means and evolves according to the time-dependent Gross-Pitaevskii equation, this singularity in the solution will be present all the time. It is because the

Gross-Pitaevskii equation is regular and the vortex singularities are rigid.⁹ This is the reason why the estimation of lifetime, given by Eq. (24) of Ref. 5 is not valid.

To conclude, we have shown that half-quantum vortices remain stable in the presence of TE-TM splitting of polariton band but their texture becomes warped—the polarization and

phase angles depend nonlinearly on the azimuthal angle. We calculated the warping effect far away from the half-vortex core.

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⁶Note that, contrary to Ref. 5, we write cylindrical coordinates as (r, ϕ) . We use θ to denote the common phase of the condensate

order parameter.

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⁸Note that the circular components become decoupled at $r \rightarrow 0$ since (in the given example) $\psi_- \propto z$ and $\partial^2 \psi_- / \partial z^2 = 0$. This is why the warping of streamlines is expected to decrease with decreasing r and disappear within the half-vortex core.

⁹Such singularities or dislocations have been discussed for the case of linear singular optics by J. F. Nye and M. V. Berry, *Proc. R. Soc. London, Ser. A* **336**, 165 (1974); J. F. Nye, *ibid.* **389**, 279 (1983).